

Systems Modeling With Stainless

Romain Ruetschi

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Abstract

We present three orthogonal approaches to the verification of non-trivial programs, and systems of medium complexity, such as evaluators for domain specific languages, small actor systems, and implementations of bi-party communication protocols. We first discuss the design, implementation and semantics of a partial symbolic evaluation procedure for PureScala programs, as well as the challenges we faced to ensure termination of said procedure. Next, we define a model for actor systems which we implement as a library for Stainless, and explain how to verify that a global invariant of an actor system is preserved between each step of execution, with supporting examples. At last, we show how the addition of linear types to the type system of Stainless allows us to safely model bi-party communication protocols expressed as session types in PureScala, with a supporting example.

Master Thesis Project under the supervision of
Prof. Viktor Kuncak & Dr. Jad Hamza
Lab for Automated Reasoning and Analysis LARA - EPFL



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1 Introduction

This project explores three orthogonal approaches to the verification of non-trivial programs, and systems of medium complexity, such as evaluators for domain specific languages, small actor systems, and implementations of bi-party communication protocols. While the techniques that we develop in this report can each stand on their own, we nonetheless briefly discuss, in each section, how a particular technique could potentially apply to one of the problem studied in another section. In Section 2, we introduce Stainless, a verification system for programs expressed in PureScala, a subset of the Scala programming language. Section 3 presents the design, and implementation of a partial symbolic evaluation procedure for PureScala programs. We then discuss its semantics, as well as the challenges we faced to ensure the termination of said procedure, when it is warranted. At last, we showcase the versatile abilities of the partial symbolic evaluator by applying it to non-trivial programs in two different contexts. In Section 4, we define a model for actor systems which we implement as a library for Stainless. We then explain how one would go about verifying that a global invariant of an actor system is preserved between each step of execution. We subsequently implement a few small systems using this library, and verify for each system that the appropriate invariant is indeed preserved. At last, in Section 5, we show how the addition of linear types to the type system of Stainless allows us to model bi-party communication protocols expressed as session types in PureScala, using the encoding provided by an existing Scala library, while statically enforcing some safety properties which were missing from the original library.

2 Program Verification with Stainless

Stainless [1] is a verification system for higher-order programs expressed in a strict subset of the Scala programming language called *PureScala*¹, derived from an earlier system called Leon [2]. In particular, Stainless can²

- verify statically that a program conforms to a given specification and that it cannot crash at run-time
- provide useful counterexamples when an implementation does not satisfy its specification
- verify that a program will terminate on all inputs

It supports both purely functional and imperative programs [3, 4].

The PureScala language, while not as expressive as Scala itself, supports a good chunk of its features, such as: recursive functions, conditional expressions, pattern matching, algebraic data types, methods, generics, first-class functions, sets, maps, mutable arrays, mutable variables, and while loops. Functions and methods can be annotated with pre- and a post-conditions, using standard Scala syntax. The verifier then extracts these specifications from the program, and attempts to automatically prove that the implementation of the program matches these specifications. It does so by generating *verification conditions* from both the specifications and the implementation, which are then fed to Inox³, a powerful solver for higher-order functional programs. Inox itself relies on various *Satisfiability Modulo Theories* [5] solvers to eventually check the validity of the given verification conditions.

```
def factorial(n: BigInt): BigInt = {
  require(n >= 0)
  if(n == 0) {
    BigInt(1)
  } else {
    n * factorial(n - 1)
  }
} ensuring(res => res >= 0)
```

Listing 1: Factorial in PureScala

¹<https://github.com/epfl-lara/stainless/blob/master/core/src/sphinx/purescala.rst>

²<https://github.com/epfl-lara/stainless/blob/master/core/src/sphinx/intro.rst>

³<https://github.com/epfl-lara/inox>

For example, given the PureScala program in Listing 1, Stainless generates the following two verification conditions:

- One corresponding to the *postcondition* of the `factorial` function:

```
(n >= 0 && n != 0) ==> (n - 1 >= 0)
```

- Another one corresponding the check for the satisfiability of `factorial`'s precondition at the recursive call `factorial(n - 1)`:

```
(n >= 0) ==> {
  val res: BigInt = if (n == 0) {
    1
  } else {
    n * factorial(n - 1)
  }
  res >= 0
}
```

The validity of the two VCs is checked by Inox, which then informs Stainless of the result, yielding the following output in the console:

```
factorial    precondition. (call factorial(n - 1))    valid    ...    0.195
factorial    postcondition                               valid    ...    0.124
```

Suppose we instead wrote the following incorrect implementation of the factorial function:

```
def factorial(n: BigInt): BigInt = {
  require(n >= 0)
  if(n == 0) {
    BigInt(1)
  } else {
    n * factorial(n - 2) // <- typo
  }
} ensuring(res => res >= 0)
```

The check for the precondition of the call to `factorial` would then be invalid, and Stainless will report a counter-example:

```
- Result for 'precond. (call factorial(n - 2))' VC for factorial @10:11:
  => INVALID
Found counter-example:
  n: BigInt -> 1
```

At last, if Inox is unable to prove either the validity or the invalidity of a verification condition in a (user-defined) given amount of time, Stainless will report `UNKNOWN`, which often means that additional lemmas and/or more specific contracts are needed.

3 Symbolic Partial Evaluation of PureScala Programs

3.1 Motivation

Let's now consider the following PureScala program:

```
def foldLeft[A, B](list: List[A], z: B)(f: (B, A) => B): B = list match {
  case Nil      => z
  case x :: xs => foldLeft(xs, f(z, x))(f)
}

def insert[A, B](kvs: List[(A, B)], map: Map[A, B]) = {
  foldLeft(kvs, map) {
    case (acc, (k, v)) if !acc.contains(k) => acc.updated(k, v)
    case (acc, _)                        => acc
  }
}

def test(map: Map[String, Int], x: Int, y: Int): (Int, Map[String, Int]) = {
  require(!map.contains("a") && map.contains("b"))

  val xs = ("a", x) :: ("b", y) :: Nil
  val res = insert(xs, map)

  res("a") == x && res("b") == map("b")
} holds
```

Listing 2: Example program

Although this program is correct, Stainless is unfortunately unable to prove the validity of verification condition corresponding to `test`'s postcondition. To remedy this problem, one could refine the program by adding contracts to the `foldLeft` and `insert` functions, or introduce additional lemmas. While this is a reasonable way of going at the problem, we will discuss in this chapter an alternative way of proving such programs correct. The main insight we will make use of is that, although none of the arguments to `insert` are fully known within the `test` function, the knowledge we have of `map` from `test`'s precondition combined with the known structure of `xs` is enough to *partially evaluate* in a *symbolic* way the body of the function, yielding a much simpler expression which in turn makes the postcondition much easier to verify.

Before we discuss this idea in greater details, let's look at how one would "manually" prove correct the above program:

1. Let's consider the call to `insert` in the test function:

```
insert(xs, map)
```

2. We start by inlining the definition of `xs`:

```
insert(("a", x) :: ("b", y) :: Nil, map)
```

3. We then unfold the definition of `insert`:

```
foldLeft(("a", x) :: ("b", y) :: Nil, map) {
  case (acc, (k, v)) if !acc.contains(k) => acc.updated(k, v)
  case (acc, _)                          => acc
}
```

4. We now unfold the definition of `foldLeft`:

```
val f = {
  case (acc, (k, v)) if !acc.contains(k) => acc.updated(k, v)
  case (acc, _)                          => acc
}

(("a", x) :: ("b", y) :: Nil) match {
  case Nil      => map
  case x :: xs => foldLeft(xs, f(z, x))(f)
}
```

For legibility, we have assigned the supplied match expression to a variable instead of inlining it straight away.

5. We select the second case of the pattern match:

```
val f = {
  case (acc, (k, v)) if !acc.contains(k) => acc.updated(k, v)
  case (acc, _)                          => acc
}

foldLeft("b" -> y :: Nil, f(map, ("a", x)))(f)
```

3 Symbolic Partial Evaluation of PureScala Programs

6. We now evaluate the first two arguments of the call to `foldLeft`. The first one stays as is as it is already in normal form. The second argument `f(map, ("a", x))` becomes:

```
(map, ("a", x)) match {  
  case (acc, (k, v)) if !acc.contains(k) => acc.updated(k, v)  
  case (acc, _) => acc  
}
```

7. Recall the precondition of the `test` function: `!map.contains("a") && map.contains("b")`. When we inline the scrutinee into the guard of the first case, we obtain `!map.contains("a")`, which by the aforementioned precondition we can deduce is `true`. We thus select the first case, and inline the scrutinee within it, yielding:

```
map.updated("a", x)
```

8. We are now left with this expression:

```
foldLeft("b" -> y :: Nil, map.updated("a", x))(f)
```

9. We can now unfold `foldLeft` one more time:

```
(("b", y) :: Nil) match {  
  case Nil => map.updated("a", x)  
  case x :: xs => foldLeft(xs, f(z, x))(f)  
}
```

10. Once again, we recurse in the second case of the match expression:

```
foldLeft(Nil, f(map.updated("a", x), ("b", y)))(f)
```

11. We now unfold the definition of `f` in `f(map.updated("a", x), ("b", y))`, yielding:

```
(map.updated("a", x), ("b", y)) match {  
  case (acc, (k, v)) if !acc.contains(k) => acc.updated(k, v)  
  case (acc, _) => acc  
}
```

12. Recall once again the precondition of the `test` function: `!map.contains("a") && map.contains("b")`. This time, the guard reads as `!map.contains("b")`, which by precondition we can deduce is `false`. We thus select the second case, and inline the scrutinee within it, yielding:

```
map.updated("a", x)
```

13. We are now left with the following expression:

```
foldLeft( Nil, map.updated("a", x))(f)
```

14. We can now unfold `foldLeft` one last time:

```
Nil match {
  case Nil      => map.updated("a", x)
  case x :: xs => foldLeft(xs, f(z, x))(f)
}
```

15. This time, we recurse in the first case, which leaves us with:

```
map.updated("a", x)
```

16. At last, we substitute this expression for the call to `insert` within `test`'s body, resulting in:

```
def test(map: Map[String, Int], x: Int, y: Int): (Int, Map[String, Int]) = {
  require(!map.contains("a") && map.contains("b"))

  val xs = ("a", x) :: ("b", y) :: Nil
  val res = map.updated("a", x)

  res("a") == x && res("b") == map("b")
} holds
```

17. This expression cannot be reduced further, as `map` is abstract and `updated` is a special language construct which can only be evaluated when applied to a concrete, finite map.
18. If one knows the semantics of `Map`, it becomes fairly obvious that the postcondition is valid, for any initial value of `map`, `x`, and `y`.

We want to draw the reader's attention to the fact that the actual values `x` and `y` did not play any role in this process. Moreover, while the exact content of the `Map` underlying the `map` variable was unknown, we knew enough about it from the precondition of `test` to successfully reduce the program to a simple expression. We formalize this whole process in the next section.

3.2 Symbolic Partial Evaluation

Let's first get a couple of definitions out of the way:

- *Partial evaluation* [6] is the process of executing a program in the presence of partial inputs, effectively specializing it program that is more performant or easier to reason about than the initial one.

3 Symbolic Partial Evaluation of PureScala Programs

- *Symbolic evaluation* [7,8] is the process of gathering knowledge about symbolic values in a program by executing it, in order to verify whether or not a given program satisfies some properties, such as the absence of divisions by zero, or the preservation of some invariant during execution [9].
- A *path condition* is a quantifier-free formula in conjunctive normal form which encodes the knowledge about the symbolic values encountered so far that the evaluator has gathered along the way.

Combining both techniques into a single evaluation procedure, ie. a procedure which partially evaluates a program with symbolic inputs while maintaining a *path condition*, has been shown to yield a much more powerful evaluator than each procedure on its own, and to increase both performance and reasoning abilities of verifications systems [10]. We call such a procedure a *symbolic partial evaluator*. The key benefit of this technique is that, whenever the evaluator encounters a boolean expression while evaluating a program, it can make use of the path condition to hopefully determine whether the expression is **true** or **false**.

Listing 3 shows a simple program annotated with the current path condition at various points of execution. This path condition enables the partial evaluator to reduce the whole expression to the **else** branch of the conditional expression because, under the path condition at step 2, the condition $x > 0$ is provably **false**.

```
// Expression before evaluation:
{
  // 1. PC = true
  assume(x <= 0)
  // 2. PC = x <= 0
  if (x > 0) {
    // 3. PC = x <= 0 && x > 0
    x
  } else {
    // 4. PC = x <= 0 && !(x > 0)
    -x
  }
}

// Resulting expression after evaluation:
{
  assume(x <= 0)
  bar(-x)
}
```

Listing 3: Program annotated with path conditions and result of its symbolic partial evaluation

3.3 Semantics

We subsequently denote the path condition by Δ , an boolean expression e that evaluates to **true** under Δ by $e \in \Delta$, and a variable v that is bound to an expression e under Δ by $v \mapsto_{\Delta} e$. We will not delve into the details of how to decide whether a given condition is satisfiable under some path condition, but a reasonable although much too simplistic approximation would be to maintain a set of expressions and simply testing for membership. We denote the fact that an expression e evaluates to an expression e' under a path condition Δ by $\llbracket e; \Delta \rrbracket \longrightarrow e'$. Given c, v two expressions, and x a variable, we note $\Delta \cup c$ the addition of c as a conjunct to the path condition Δ , and $\Delta \uplus x \mapsto v$ the addition of the binding `val x = v`. At last, the fact that a function invocation f can be safely unfolded under path condition Δ is captured by $f \Downarrow \Delta$. Figure 1 shows the operational semantics followed by our implementation, expressed in those terms. We informally discuss the most interesting of these rules below.

Evaluation of Boolean expressions

To determine the value of an expression c of type `Boolean`, the evaluator proceeds as follows:

1. If c is a boolean literal, the evaluator returns c .
2. Otherwise, it checks whether the formula $\Delta_{CNF} \implies c$ is valid, where Δ_{CNF} is the CNF formula corresponding to the path condition Δ :
 - (a) If it is valid, the evaluator returns **true**.
 - (b) Otherwise, it checks whether the formula $\Delta_{CNF} \implies \neg c'$ is valid, in which case it returns **false**.
 - (c) If none are valid, the expression goes through the other evaluation rules based on its structure.

Evaluation of conditional expressions

When encountering a conditional expression $E = \text{if } (c) t \text{ else } e$, under a path condition Δ , the evaluator proceeds as follows:

1. It partially evaluates c , resulting in an expression c' .
2. If c' is the boolean literal **true**, it recurses into the **then** branch of the conditional expression under the path condition $\Delta \cup c'$.
3. If c' is the boolean literal **false**, it recurses into the **else** branch under the path condition $\Delta \cup \neg c'$.
4. If c' is not a literal, it evaluates both branches under their respective path conditions. We denote the result of their evaluation respectively by t' and e' . It then returns the expression `if (c') t' else e'`.

This procedure is formalized in rules 10, 11, and 12 of Figure ??.

Evaluation of pattern matches

Pattern matches are converted to an equivalent conditional expression with explicit type checks, casts, and field accesses, before being fed back to the evaluator. The corresponding rule (21) can be found in Figure ??.

Evaluation of function invocations

Finding good rules for the evaluation of function invocations, and more specifically recursive one, was by far the most challenging and subtle aspect of this work. Indeed, the naive approach of unfolding every function call that is encountered during evaluation send the evaluator into an infinite loop. Instead, the evaluation of a function call proceeds as follows:

1. The arguments are recursively evaluated.
2. The precondition of the function is then checked against the evaluated arguments. If is invalid, the evaluator stops there and replace the original arguments to the invocation by their counterpart computed in the step 1.
3. If the function that is invoked is not recursive, the invocation is unfolded, and the evaluator recurses over the resulting expression.
4. If the function is recursive, some more care is needed. Indeed, if one were to unfold every calls to a recursive function, this would often send the evaluator into an infinite loop, even when the function has been deemed terminating. To understand why, let's consider the following example:

```
def fact(n: Int): Int = {
  require(n >= 1)
  if (n == 1) 1 else n * fact(n - 1)
}

def test(n: Int): Int = fact(n)
```

If we were to unfold `fact` in function `test` and continue evaluation from there, because we do not know anything about the value of `n`, the evaluator would recursive into both branches of the conditional expression. In the `else` branch, the recursive call to `fact` would itself be unfolded. Because we still do not know anything about the value of `n - 1`, both branches of the expression resulting from unfolding `fact` once more would also be explored, and so on. This obviously send the evaluator into an infinite loop, and prevent termination of the evaluation. On the other hand, if we knew more about `n`, eg. that `n >= 3`, the evaluator would be able to unfold `fact` a number of times already, eg. twice resulting in `n * n * fact(n - 2)`.

In other words, we only want to unfold an invocation to a recursive function if we can determine, under the current path condition, whether or not a subsequent, valid recursive call will definitely be reached or not. We denote such a function invocation by $f(e_1, \dots, e_2) \Downarrow \Delta$. We discuss the impact of this heuristic on termination in more details in Section 3.4.

5. If the function invocation cannot be productively unfolded, the evaluator stops there and replace the original arguments to the invocation by their counterpart computed in step 1.

The corresponding rules (25, 26, and 27) can be found in Figure ??.

$$\frac{v \mapsto_{\Delta} e}{\llbracket v; \Delta \rrbracket \rightarrow e} \quad (1)$$

$$\frac{e \in \Delta}{\llbracket e; \Delta \rrbracket \rightarrow \text{true}} \quad (2)$$

$$\frac{\neg e \in \Delta}{\llbracket e; \Delta \rrbracket \rightarrow \text{false}} \quad (3)$$

$$\frac{\llbracket e; \Delta \rrbracket \rightarrow e'}{\llbracket \lambda x_1, \dots, x_n. e; \Delta \rrbracket \rightarrow \lambda x_1, \dots, x_n. e'} \quad (4)$$

$$\frac{\llbracket \neg l \ || \ r; \Delta \rrbracket \rightarrow e'}{\llbracket l \ ==> \ r; \Delta \rrbracket \rightarrow e'} \quad (5)$$

$$\frac{\llbracket e_1; \Delta \rrbracket \rightarrow \text{false}}{\llbracket e_1 \ \&\& \dots \ \&\& \ e_n; \Delta \rrbracket \rightarrow \text{false}} \quad (6)$$

$$\frac{\llbracket e_1; \Delta \rrbracket \rightarrow e'_1 \quad \llbracket e_2 \ \&\& \dots \ \&\& \ e_n; \Delta \rrbracket \rightarrow e'}{\llbracket e_1 \ \&\& \dots \ \&\& \ e_n; \Delta \rrbracket \rightarrow e'_1 \ \&\& \ e'} \quad (7)$$

$$\frac{\llbracket e_1; \Delta \rrbracket \rightarrow \text{true}}{\llbracket e_1 \ || \ \dots \ || \ e_n; \Delta \rrbracket \rightarrow \text{true}} \quad (8)$$

$$\frac{\llbracket e_1; \Delta \rrbracket \rightarrow e'_1 \quad \llbracket e_2 \ || \ \dots \ || \ e_n; \Delta \rrbracket \rightarrow e'}{\llbracket e_1 \ || \ \dots \ || \ e_n; \Delta \rrbracket \rightarrow e'_1 \ || \ e'} \quad (9)$$

$$\frac{\llbracket c; \Delta \rrbracket \rightarrow \text{true}}{\llbracket \text{if } (c) \ t \ \text{else } e; \Delta \rrbracket \rightarrow \llbracket t; \Delta \cup c \rrbracket} \quad (10)$$

$$\frac{\llbracket c; \Delta \rrbracket \rightarrow \text{false}}{\llbracket \text{if } (c) \ t \ \text{else } e; \Delta \rrbracket \rightarrow \llbracket e; \Delta \cup \neg c \rrbracket} \quad (11)$$

$$\frac{\llbracket c; \Delta \rrbracket \rightarrow c' \quad \llbracket t; \Delta \cup c' \rrbracket \rightarrow t' \quad \llbracket e; \Delta \cup \neg c' \rrbracket \rightarrow e'}{\llbracket \text{if } (c) \ t \ \text{else } e; \Delta \rrbracket \rightarrow \text{if } (c') \ t' \ \text{else } e'} \quad (12)$$

Figure 1: Operational semantics of the symbolic partial evaluator

$$\frac{\llbracket p; \Delta \rrbracket \rightarrow \text{true} \quad \llbracket e; \Delta \rrbracket \rightarrow e'}{\llbracket \text{assume}(p); e; \Delta \rrbracket \rightarrow e'} \quad (13)$$

$$\frac{\llbracket p; \Delta \rrbracket \rightarrow \text{false} \quad \llbracket e; \Delta \rrbracket \rightarrow e'}{\llbracket \text{assume}(p); e; \Delta \rrbracket \rightarrow \text{assume}(\text{false}); e'} \quad (14)$$

$$\frac{\llbracket p; \Delta \rrbracket \rightarrow p' \quad \llbracket e; \Delta \cup p' \rrbracket \rightarrow e'}{\llbracket \text{assume}(p); e; \Delta \rrbracket \rightarrow \text{assume}(p'); e'} \quad (15)$$

$$\frac{T_2: \text{ADTType} \quad \neg \text{isSort}(T_2)}{\llbracket C(T_1, a_1, \dots, a_n). \text{isInstanceOf}[T_2]; \Delta \rrbracket \rightarrow T_1.\text{id} == T_2.\text{id}} \quad (16)$$

$$\frac{T: \text{ADTType} \quad \text{isSort}(T)}{\llbracket e.\text{isInstanceOf}[T]; \Delta \rrbracket \rightarrow \text{true}} \quad (17)$$

$$\frac{T: \text{ADTType} \quad \llbracket e; \Delta \rrbracket \rightarrow e' \quad \text{isInstanceOf}(e', T, \Delta) == \text{Some}(b)}{\llbracket e.\text{isInstanceOf}[T]; \Delta \rrbracket \rightarrow b} \quad (18)$$

$$\frac{T: \text{ADTType} \quad \llbracket e; \Delta \rrbracket \rightarrow e' \quad \text{isInstanceOf}(e', T, \Delta) == \text{None}}{\llbracket e.\text{isInstanceOf}[T]; \Delta \rrbracket \rightarrow e'.\text{isInstanceOf}[T]} \quad (19)$$

$$\frac{T: \text{ADTType}}{\llbracket e.\text{asInstanceOf}[T]; \Delta \rrbracket \rightarrow \llbracket e; \Delta \rrbracket.\text{asInstanceOf}[T]} \quad (20)$$

$$\frac{e.\text{getType} = \text{MatchExpr} \quad \llbracket \text{matchToIfThenElse}(e); \Delta \rrbracket \rightarrow e'}{\llbracket e; \Delta \rrbracket \rightarrow e'} \quad (21)$$

Figure 1: Operational semantics of the symbolic partial evaluator

$$\overline{\llbracket \text{let } x:T = v \text{ in } e; \Delta \rrbracket} \longrightarrow \llbracket e[x/v]; \Delta \rrbracket \quad (22)$$

$$\frac{\llbracket f; \Delta \rrbracket \longrightarrow \lambda x_1:T_1, \dots, x_n:T_n. b \quad \llbracket e_i; \Delta \rrbracket \longrightarrow e'_i, i \in \{1 \dots n\}}{\llbracket f(e_1, \dots, e_n); \Delta \rrbracket \longrightarrow \llbracket b[x_1/e'_1, \dots, x_n/e'_n]; \Delta \rrbracket} \quad (23)$$

$$\frac{\llbracket f; \Delta \rrbracket \longrightarrow f' \quad \llbracket e_i; \Delta \rrbracket \longrightarrow e'_i, i \in \{1 \dots n\}}{\llbracket f(e_1, \dots, e_n); \Delta \rrbracket \longrightarrow f'(e'_1, \dots, e'_n)} \quad (24)$$

$$\frac{\neg f.\text{isRecursive} \quad f.\text{params} = \langle x_1, \dots, x_n \rangle \quad \llbracket e_i; \Delta \rrbracket \longrightarrow e'_i, i \in \{1 \dots n\} \quad \llbracket f.\text{pre}(e'_1, \dots, e'_n); \Delta \rrbracket \longrightarrow \text{true}}{\llbracket f(e_1, \dots, e_n); \Delta \rrbracket \longrightarrow \llbracket f.\text{body}[x_1/e'_1, \dots, x_n/e'_n]; \Delta \rrbracket} \quad (25)$$

$$\frac{f.\text{params} = \langle x_1, \dots, x_n \rangle \quad \llbracket e_i; \Delta \rrbracket \longrightarrow e'_i, i \in \{1 \dots n\} \quad \llbracket f.\text{pre}(e'_1, \dots, e'_n); \Delta \rrbracket \longrightarrow \text{true} \quad f(e'_1, \dots, e'_n) \Downarrow \Delta}{\llbracket f(e_1, \dots, e_n); \Delta \rrbracket \longrightarrow \llbracket f.\text{body}[x_1/e'_1, \dots, x_n/e'_n]; \Delta \rrbracket} \quad (26)$$

$$\frac{\llbracket e_i; \Delta \rrbracket \longrightarrow e'_i, i \in \{1 \dots n\}}{\llbracket f(e_1, \dots, e_n); \Delta \rrbracket \longrightarrow f(e'_1, \dots, e'_n)} \quad (27)$$

$$\frac{(\text{cons}, (e_1, \dots, e_n)) = \text{deconstruct}(e) \quad \llbracket e_i; \Delta \rrbracket \longrightarrow e'_i, i \in \{1 \dots n\}}{\llbracket e; \Delta \rrbracket \longrightarrow \text{cons}(e'_1, \dots, e'_n)} \quad (28)$$

Figure 1: Operational semantics of the symbolic partial evaluator

3.4 Termination

When the partial evaluator is invoked over a program that is not provably terminating, there are no guarantees that the evaluation procedure will terminate as well. If the program is provably non-terminating, such as the one in Figure 4, then the evaluator will not terminate either. Indeed, in the current state, checking for program termination using Stainless’s powerful termination checker during evaluation is impractical for performance reasons. While it would certainly be possible to perform termination checking before the partial evaluator kicks in, and then only attempt to evaluate functions which have been proved terminating, we for now simply suggest to only enable partial evaluation on functions which have been proved terminating by Stainless, by running the termination checker manually, as is the recommended practice when doing verification as well. As an additional measure, we have bounded the maximum number of evaluation steps that can be performed by the evaluator. This ensure that, should one still run the evaluator on a non-termination function, Stainless will not hang or crash.

```
def bar(xs: List[BigInt]): List[BigInt] = {
  if (xs.isEmpty) Nil() else bar(Cons(xs.head, xs))
}

@partialEval
def test = {
  bar(List(1)) == Nil()
} holds
```

Listing 4: Example of non-terminating program

Moreover, we argue that, when ran over a program that Stainless deems terminating, the evaluator terminates as well. We note that, at point in the pipeline, the only source of non-termination are calls to recursive functions. As such, because the evaluator will only unfold a function invocation when the precondition is satisfied, and when any of the following two statement is true:

- (a) The unfolded expression does not contain a recursive call
- (b) Otherwise, it is possible to determine under the current path condition whether a recursive call will be reached or not

The understand why the second property is important, consider the program in Listing 5. If the evaluator only looked at the outermost conditional statement to determine whether the call to `tricky` in `test` could be unfolded in productive way, it would end up in an infinite loop. Indeed, because `n` is greater than 0 under the path condition, the `then` branch of the outermost `if` will be selected, and evaluation will continue recursively from there. Because we don’t know anything about `xs`, the evaluator will then speculatively evaluate both branches of the `match` expression, and will thus encounter another recursive call, which it will deem worthy of unfolding because, by the same reasoning, the outermost `if` can be reduced to its `then` branch. On the

other hand, property (b) will forbid the evaluator to unfold `tricky` because it is not possible to deduce, under the current path condition, whether or not the recursive call will be reached.

On the other hand, if we supply `tricky` with a partially known list, such as in the function `test2`, `tricky` will still be unfolded until the function is eventually applied to the abstract list `xs`. While this no formal argument, we believe that these conditions are sufficient to ensure termination of evaluation of provably terminating programs.

```
def tricky(n: Int, xs: List[Int]): Int = {
  if (n > 0) {
    xs match {
      case Nil() => n
      case Cons(x, xs) => x + tricky(n, xs)
    }
  } else {
    0
  }
}

@partialEval
def test1(n: Int, xs: List[Int]) = {
  require(n > 0)
  tricky(n, xs) // tricky(n, xs)
}

@partialEval
def test2(n: Int, xs: List[Int]) = {
  require(n > 0)
  tricky(n, Cons(1, Cons(2, Cons(3, xs)))) // 1 + (2 + (3 + trick(n, xs)))
}
```

Listing 5: Program requiring some care during evaluation

Regarding termination checking itself, we have ensured that the partial evaluator cannot be enabled when checking termination in Stainless, as doing so could prevent the termination checker to itself terminate, or even yield incorrect results.

3.5 Implementation

We have implemented the symbolic partial evaluator described above within Stainless⁴ in about 900 lines of code, leveraging Inox to check for validity of the formulas underlying the path condition. In order to invoke the partial evaluator on a function definition, a user must supply the new `-partial-eval` flag and annotate the function with the `@partialEval` annotation. Any such function will then be partially evaluated, and its body will be substituted by the resulting expression. Enabling the `partial-eval` debug section instructs Stainless to print the result of

⁴<https://github.com/romac/stainless/tree/sym-partial-eval>

the evaluation. This effectively enable users to use Stainless as a symbolic partial evaluator for PureScala program.

3.6 Case Studies

3.6.1 Key-Value Store Algebra

In Listing 8, we define a monadic domain-specific language to manipulate a key-value store, along with a tracing interpreter which maps the operations over a PureScala `Map`. A simple program which makes use of this DSL to insert a list of concrete values into the store is shown in Listing 7. Without additional lemmas, Stainless is unfortunately unable to verify the postcondition of the `result` function. On the other hand, turning on the partial evaluator turns the associated verification condition into a much more concrete form, which in turns allow the verification to promptly go through. We will note that, although the exact amount of *fuel* we give to the interpreter is unknown, the precondition of the `result` function constrains it to be greater than 10. Because the program expressed in that DSL requires less than 10 steps to be fully interpreted, the evaluator is able to reduce it to normal form.

```

val (res, trace) = (
  map.updated("foo", "bar").updated("toto", "tata")("foo"),
  Get("foo") :: Put("toto", "tata") :: Put("foo", "bar") :: init
)

res == Some("bar") &&
trace.take(3) == Get("foo") :: Put("toto", "tata") :: Put("foo", "bar") :: Nil()

```

Listing 6: Partially evaluated verification condition

3.6.2 1st Futamura Projection

Let us consider the case where one were to define a language, implement an interpreter for it in PureScala, and then write a program in this language. One could then partially evaluate the interpreter applied to the program, while keeping the inputs and I/O operations abstract. The resulting expression could then be seen as a "compiled" version of the original program, represented as a PureScala program, which can be linked to a small runtime and compiled to a lower-level language using the infrastructure provided by the Scala language itself, PureScala being a strict subset of Scala.

The technique of partially evaluating an interpreter over a known program is known as the *1st Futamura projection* [11], and can theoretically be applied to the partial evaluator itself, yielding a compiler (the 2nd projection) or even a converter from any interpreter to a compiler (3rd projection). Listing 10 features an example involving an interpreter for a simple language with variables, arithmetic expressions and an I/O operation.

```

def insert(kvs: List[(String, String)])(after: Op): Op = kvs match {
  case Nil() => after
  case Cons((k, v), rest) => put(k, v) { () => insert(rest)(after) }
}

val xs = List("foo" -> "bar", "toto" -> "tata")
val program = insert(xs) {
  get("foo") { foo =>
    pure(foo)
  }
}

@partialEval
def result(map: Map[String, String], init: List[Label], fuel: BigInt) = {
  require(fuel > 10)
  interpret(program)(map, init, fuel)
} ensuring { case (res, trace) => prop(res, trace) }

@inline
def prop(res: Option[String], trace: List[Label]) = {
  res == Some("bar") &&
  trace.take(3) == List(
    Label.Get("foo"),
    Label.Put("toto", "tata"),
    Label.Put("foo", "bar")
  )
}

```

Listing 7: Example program

3.7 Conclusion

We have presented, formalized and implemented a symbolic partial evaluator for PureScala programs which:

1. enhances the verification capability of Stainless by allowing the system to reason about programs at a semantic level closer to those of PureScala, effectively simplifying the verification conditions then generated by the system. This in turn helped us reason about a tracing interpreter for a simple algebra over a key-value store.
2. effectively turns Stainless into a symbolic partial evaluator for PureScala programs, thanks to user-land annotations which allow users to selectively simplify parts of their programs. This allowed us to turn an interpreter for a simple programming language into a compiler from that language to PureScala.

```

sealed abstract class Label
object Label {
  case class Get(key: String) extends Label
  case class Put(key: String, value: String) extends Label
}

sealed abstract class Op
case class Pure(value: Option[String]) extends Op
case class Get(key: String, next: Option[String] => Op) extends Op
case class Put(key: String, value: String, next: () => Op) extends Op

def get(key: String)(next: Option[String] => Op): Op = Get(key, next)
def put(key: String, value: String)(next: () => Op): Op = Put(key, value, next)
def pure(value: Option[String]): Op = Pure(value)

def interpret(op: Op)(kv: Map[String, String], trace: List[Label], fuel: BigInt) = {
  require(fuel >= 0)
  decreases(fuel)

  op match {
    case Pure(value) =>
      (value, trace)

    case Get(key, next) if fuel > 0 =>
      interpret(next(kv.get(key)))(
        kv,
        Label.Get(key) :: trace,
        fuel - 1
      )

    case Put(key, value, next) if fuel > 0 =>
      interpret(next())(
        kv.updated(key, value),
        Label.Put(key, value) :: trace,
        fuel - 1
      )

    case _ =>
      (None(), trace)
  }
}

```

Listing 8: Key-value store implementation

3.8 Further Work

Nondeterministic Evaluator

The evaluator described in this chapter is *deterministic*, in the sense that its evaluation function maps one expression to exactly one expression in a deterministic way. We believe that it could be worthwhile to investigate the potential use cases of a so-called *non-deterministic* evaluator. That is to say, an evaluator which, when given a single expression, would return potentially multiple results. For example, when the evaluator would be applied to an expression $s = \text{if } (c) \ t \ \text{else } e$ whose condition c it is not able to evaluate to a boolean literal, it would return all the possible expressions resulting from first evaluating the condition and both branches nondeterministically. We show some pseudo-code in Listing 11.

One potential use case that we see for such an evaluator, is the user-land implementation of a symbolic *dynamic partial order reduction* algorithm for transition-based systems, such as actor systems. Indeed, our hope is that such an evaluator could be used to compute the set of concrete transitions that would result from taking a step at a some partially symbolic state.

```

sealed trait Expr
case class Var(name: String) extends Expr
case class Num(value: Int) extends Expr
case class Add(l: Expr, r: Expr) extends Expr
case class Mul(l: Expr, r: Expr) extends Expr
case class Rand(max: Expr) extends Expr

case class Context(bindings: List[(String, Expr)]) {
  def contains(name: String): Boolean =
    apply(name).isDefined

  def apply(name: String): Option[Expr] =
    bindings.find(_._1 == name).map(_._2)
}

implicit val state = Random.newState

@extern
def random(max: Int): Int = {
  Random.nextInt(max)
}

case class Error(msg: String)
def interpret(expr: Expr, ctx: Context): Either[Error, Int] = {
  expr match {
    case Num(value) => Right(value)

    case Var(name) => ctx(name) match {
      case None() => Left(Error("Unbound variable: " + name))
      case Some(value) => interpret(value, ctx)
    }

    case Add(l, r) => for {
      le <- interpret(l, ctx)
      re <- interpret(r, ctx)
    } yield le + re

    case Mul(l, r) => for {
      le <- interpret(l, ctx)
      re <- interpret(r, ctx)
    } yield le * re

    case Rand(max) =>
      interpret(max, ctx).map(random(_))
  }
}

```

Listing 9: Implementation an interpreter for a simple language


```

val program: Expr = Mul(Num(10), Add(Var("x"), Rand(Num(42))))

@partialEval
def compiled(ctx: Context): Int = {
  require(ctx contains "x")
  interpret(program, ctx).get.           // 10 * (ctx("x") + random(42))
}

@partialEval
def test(y: Int) = {
  val ctx = Context(Map("y" -> Num(y)))
  interpret(program, ctx)                // Left(Error("Unbound variable: x"))
} ensuring { _.isLeft }

```

Listing 10: "Compilation" of a simple program

```

def ndEval(expr: Expr, pc: PC): Stream[Expr] = {
  case IfExpr(c, t, e) =>
    for {
      c <- ndEval(c, pc)
      ts = ndEval(t, pc withCond c)
      es = ndEval(e, pc withCond not(c))
    } yield if (c == true) ts else if(c == false) es else (ts ++ es)

  /* ... */
}

```

Listing 11: Pseudo-code of a non-deterministic evaluator

4 Verifying Actor Systems

4.1 Motivation

These days, the size or the throughput of the data that a typical computational system has to deal with on a regular basis requires such systems to distribute their workload over several logical processing units, such as the several logical cores that a modern CPU provides. When that is not enough, the workload is then commonly spread out over several processing units, which most often do not share an address space as they are commonly spread in one or multiple data centers. The programs which run on these system must hence be decomposed into smaller components which can all run concurrently, and therefore be spread out over multiple cores, processors, or machines. The actor model [12–14], amongst others, provides a basis for such concurrent computations. It does so by breaking up a computation into isolated units called *actors*, which communicate asynchronously by sending each other messages. Because actors do not share memory and rely on asynchronous message passing, they can easily be multiplexed over a set of logical processes or *threads*. Moreover, because of these very same properties, the actor model is also well suited to the modeling of distributed systems, as the latter are typically composed of isolated units which do not share state with the other components and rely on asynchronous message passing to communicate together.

4.2 A Simple Actor Model for Verification

Our model is loosely based on the modern definition of object-based actor systems [15], and attempts to provide an idiomatic Scala API in the style of the Akka actor library. In this framework, each actor is addressed by a *reference*, through which other actors can asynchronously send messages. Each actor has an associated *behavior*, which holds the state of the actor, if any, and determines a) the operations which will be performed upon receiving a message, and b) what is the next *behavior* to associate with its reference. A *behavior* can thus be seen as a data type holding some immutable values representing the state, and a method which takes in a message, performs some computation which might involve sending messages to other actors, and finally returns a new behavior. State at the actor level is thus effectively handled in a functional way, by returning a new behavior which holds the updated state. An *actor system* maintains a collection of one-way channels (*inboxes*) between any two actors in the system. An *inbox* is an ordered list of messages awaiting delivery, the head of the list being the next message to deliver. The system drives the execution by non-deterministically picking a non-empty inbox, taking the first message of the list, and invoking the message handler of the behavior associated with the destination reference. It then collects the messages to send, and appends them to the appropriate inboxes, and update the destination actor’s behavior with the behavior returned by the message handler.

4.3 Implementation

We have implemented the actor model described in the previous section as a PureScala library, which we detail below.

Message

In our framework, messages are modeled as constructors of the `Msg` abstract class.

```
abstract class Msg
case class Hello(name: String) extends Msg
```

Actor Reference

Each actor is associated with a persistent reference, modeled as constructors of the `ActorRef` abstract class.

```
abstract class ActorRef
case class Primary() extends ActorRef
```

In-flight Messages

In-flight messages are represented as a product of the `ActorRef` of the destination actor, and the message itself.

```
case class Packet(dest: ActorRef, payload: Msg)
```

Actor Context

When a message is delivered to an actor, the latter is provided with a context, which holds a reference to itself, and a mutable list of `Packets` to send.

```
case class ActorContext(
  self: ActorRef,
  var toSend: List[Packet]
)
```

Behavior

A behavior specifies both the current state of an actor, and how this one should process the next incoming message. In our framework, these are modeled as a subclass of the abstract class `Behavior`, which defines a single abstract method `processMsg`, to be overridden for each defined behavior. Using the provided `ActorContext`, the implementation of the `processMsg` method can both access its own reference, and register messages to be sent after the execution of the method is complete. It is also required to return a new `Behavior`

```
abstract class Behavior {
  def processMsg(msg: Msg)(implicit ctx: ActorContext): Behavior
}
```

Actor System

The state of the Actor system at a given point in time is modeled as a case class, holding the behavior associated with each actor reference, and the list of in-flight messages between any two actors.

```

case class ActorSystem(
  behaviors: CMap[ActorRef, Behavior],
  inboxes: CMap[(ActorRef, ActorRef), List[Msg]]
)

```

The `ActorSystem` class is equipped with a `step` method, which takes a pair of `ActorRef` as arguments, and is in charge of delivering the oldest message found in the corresponding inbox, and which returns the new state of the system after the aforementioned message has been processed.

```

def step(from: ActorRef, to: ActorRef): ActorSystem

```

4.3.1 Comparison with Akka and Akka Typed

Akka

A notable difference between our model and Akka⁵, is that we rely on *exactly-once* delivery of messages. We experimented with a weaker model that only relied on *at-most-once* delivery but it quickly became clear that the amount of work needed to verify the very same systems we describe below would be too much in the context of this project.

Akka Typed

While we initially set to mimic the API offered by the Akka Typed library⁶, we quickly ran into some limitations of Stainless's type system. Namely, the lack of an encoding for both existential types, and various issues with the encoding of co-/contravariant class hierarchies. While some of those issues got fixed over the last few months, we have so far been unable to provide a typed API similar in spirit to Akka Typed, and have thus settled on a somewhat weakly typed API.

4.4 Proving Invariants

After having defined an Actor system with our framework, one might want to verify that this system preserves some invariant between each step of its execution. That is to say, for an `ActorSystem` s , any two `ActorRef` n, m , and an invariant $\text{inv}: \text{ActorSystem} \rightarrow \text{Boolean}$, if $\text{inv}(s)$ holds, then $\text{inv}(s.\text{step}(n, m))$ should hold as well. We express this property more formally in Figure 2. We note that, because we are essentially doing a proof by induction over execution steps here, one needs also to ensure the invariant holds for some initial system. These two properties can be easily expressed in PureScala, as shown in Listing 12.

⁵<https://doc.akka.io/docs/akka/current/general/message-delivery-reliability.html>

⁶<https://doc.akka.io/docs/akka/2.5.4/scala/typed.html>

$$\forall s : \text{ActorSystem}, n : \text{ActorRef}, m : \text{ActorRef}. \text{inv}(s) \implies \text{inv}(s.\text{step}(n, m))$$

Figure 2: Invariant preservation property

```

def inv(s: ActorSystem): Boolean = {
  /* ... */
}

def preserveInv(s: ActorSystem, n: ActorRef, m: ActorRef): Boolean = {
  require(inv(s))
  inv(s.step(n, m))
} holds

def initial: ActorSystem = /* ... */

def initialInv: Boolean = {
  invariant(initial)
} holds

```

Listing 12: Invariant preservation theorem in PureScala

4.5 Case studies

4.5.1 Increment-based Replicated Counter

As a first and very simple case study, we will study an Actor system which models a replicated counter, which can only be incremented by one unit. This system is composed of two actors, a primary counter whose reference is `Primary()`, and a backup counter whose reference is `Backup()`. Each of these reference is associated with a behavior: the primary counter reference with an instance of `PrimaryB`, and the backup counter reference with an instance of `BackupB`, both of which hold a positive integer, representing the value of the counter. Whenever the primary actor receives the message `Inc()`, it forwards that message to the backup actor, and returns a new instance of `PrimaryB` with the counter value incremented by one. When the backup actor receives the `Inc()` message, it too returns a new instance of `BackupB` with the counter value incremented by one. The corresponding PureScala implementation can be found in Listing 13.

We now want to verify that the following properties stay valid between at execution step: the `Backup()` actor does not send itself any messages, both actors have the proper corresponding behavior, the value of the primary counter is equal to the value of the backup counter added to the number of `Inc()` messages that are yet to be delivered to the backup actor. Listing 14 presents the corresponding invariant, while Listing 15 shows the invariant preservation theorem that we want to prove.

```

case class Primary() extends ActorRef
case class Backup() extends ActorRef

case class Inc() extends Msg

case class PrimaryB(counter: BigInt) extends Behavior {
  require(counter >= 0)

  def processMsg(msg: Msg)(implicit ctx: ActorContext): Behavior = msg match {
    case Inc() =>
      Backup() ! Inc()
      PrimaryB(counter + 1)
  }
}

case class BackupB(counter: BigInt) extends Behavior {
  require(counter >= 0)

  def processMsg(msg: Msg)(implicit ctx: ActorContext): Behavior = msg match {
    case Inc() => BackupB(counter + 1)
  }
}

```

Listing 13: Increment-based replicated counter implementation

```

def invariant(s: ActorSystem): Boolean = {
  s.inboxes((Backup(), Backup())).isEmpty && {
    (s.behaviors(Primary()), s.behaviors(Backup())) match {
      case (PrimaryB(p), BackupB(b)) =>
        p.value == b.value + s.inboxes(Primary() -> Backup()).length
      case _ => false
    }
  }
}

```

Listing 14: Increment-based replicated counter invariant

```

def preserveInv(s: ActorSystem, n: ActorRef, m: ActorRef): Boolean = {
  require(invariant(s))
  invariant(s.step(n, m))
} holds

```

Listing 15: Replicated counter theorem (increment)

4.5.2 Delivery-based Replicated Counter

Listing 16 shows a variant of the previous case study, where instead of having the primary actor forward the `Inc()` message to the backup actor, the former instead sends the latter the new value.

```

case object Primary extends ActorRef
case object Backup extends ActorRef

case object Inc extends Msg
case class Deliver(c: BigInt) extends Msg

case class PrimaryB(counter: BigInt) extends Behavior {
  def processMsg(msg: Msg)(implicit ctx: ActorContext): Behavior = msg match {
    case Inc =>
      Backup ! Deliver(counter + 1)
      PrimaryB(counter + 1)

    case _ => Behavior.same
  }
}

case class BackupB(counter: BigInt) extends Behavior {
  def processMsg(msg: Msg)(implicit ctx: ActorContext): Behavior = msg match {
    case Deliver(c) => BackupB(c)
    case _          => Behavior.same
  }
}

```

Listing 16: Delivery-based Replicated counter implementation

The invariant now reads slightly differently, as can be seen in Listing 17.

```

def validBehaviors(s: ActorSystem): Boolean = {
  (s.behaviors(Primary), s.behaviors(Backup) match {
    case (p: PrimaryB, b: BackupB) => true
    case _ => false
  })
}

def invariant(s: ActorSystem): Boolean = {
  validBehaviors(s) &&
  s.inboxes(Primary -> Primary).isEmpty &&
  s.inboxes(Backup -> Backup).isEmpty &&
  noMsgsToSelf(Backup -> Primary).isEmpty &&
  {
    val PrimaryB(p) = s.behaviors(Primary)
    val BackupB(b) = s.behaviors(Backup)
    val bInbox = s.inboxes(Primary -> Backup)

    p.value >= b.value && isSorted(bInbox) && bInbox.forall {
      case Deliver(Counter(i)) => p.value >= i
      case _ => true
    }
  }
}

```

Listing 17: Delivery-based replicated counter invariant

4.6 Spawning Actors

Up until now, our framework has only been able to model Actor systems with a static topology, ie. systems where no new actors besides the ones that are statically defined can be spawned. Let's now attempt to enrich our model to account for dynamic topologies.

To this end, we modify the `ActorRef` definition to include both a name and an optional field holding a reference to its parent `ActorRef` if any. We also add a new constructor of the `ActorRef` data type, which will be assigned to actors spawned from another actor.

```
abstract class ActorRef(
  name: String,
  parent: Option[ActorRef] = None()
)

case class Child(name: String, getParent: ActorRef)
  extends ActorRef(name, Some(getParent))
```

In order for actors to spawn other actors, by specifying their name and associated initial behavior, we modify the `ActorContext` class as follows:

```
case class ActorContext(
  self: ActorRef,
  var toSend: List[Packet],
  var toSpawn: List[(ActorRef, Behavior)]
) {
  def spawn(behavior: Behavior, name: String): ActorRef = {
    val id: ActorRef = Child(name, self)
    toSpawn = toSpawn :+ (id, behavior)
    id
  }
  /* ... */
}
```

As can be seen in the listing above, the context now keeps track of the names and behaviors of the actors to be spawned, and provides a `spawn` method which is in charge of constructing the `ActorRef` of the spawned actor, storing it along with the behavior within the context, and returning the newly generated reference.

4.6.1 Case Study

Listing ?? defines a simple system with a dynamic topology, where one actor named `Primary` waits for a `Spawn` message to spawn a child actor, and change its behavior from `BeforeB` to `AfterB` in order to keep track of the reference to the child. The invariant we would to verify holds here, states that, if the `Primary` actor has behavior `BeforeB()`, then the behavior associated with the `ActorRef` of its child actor must be `Stopped`. On the other hand, if the `Primary` actor has behavior `AfterB(child)`, then the behavior associated with `child` must be `ChildB`.

```

case object Primary extends ActorRef("primary")
case object Spawn extends Msg

case class BeforeB() extends Behavior {
  def processMsg(msg: Msg)(implicit ctx: ActorContext): Behavior = msg match {
    case Spawn =>
      val child = ctx.spawn(ChildB(), "child")
      AfterB(child)
  }
}

case class AfterB(child: ActorRef) extends Behavior {
  def processMsg(msg: Msg)(implicit ctx: ActorContext): Behavior = msg match {
    case _ => Behavior.same
  }
}

case class ChildB() extends Behavior {
  def processMsg(msg: Msg)(implicit ctx: ActorContext): Behavior = msg match {
    case _ => Behavior.same
  }
}

def invariant(s: ActorSystem): Boolean = {
  s.behaviors(Primary) match {
    case BeforeB() =>
      s.isStopped(Child("child", Primary()))
    case AfterB(child) =>
      s.behaviors(child) == ChildB()

    case _ => false
  }
}

def theorem(s: ActorSystem, from: ActorRef, to: ActorRef): Boolean = {
  require(invariant(s))
  invariant(s.step(from, to))
} holds

```

4.7 Running an Actor System on Akka

While the verification of Actor systems is in itself an interesting endeavour, it is not of much use unless one is able to run these systems, potentially in a distributed environment. Thanks to the shim presented in Listing 18, it is effectively possible run an Actor system developed with our framework on top of Akka, with only a few alterations to the original program. With this shim, the ActorRef type is mapped to Akka's ActorRef, while the ActorContext now only contains the

actor's self-reference, as well as the underlying Akka context. The shim also defines an actual Akka actor, parametrized by an underlying `Behavior`, to which all messages of type `Msg` will be delegated. The `ActorSystem` class becomes abstract, and concrete subclasses need to provide it with an actual Akka system, as well as provide an implementation of its `run` method. Within this method, one can spawn new top-level actors, get a reference to those, and send them messages. Listing 19 shows such an implementation for the replicated counter described in Section 4.5.1.

```
import akka.actor

type ActorRef = actor.ActorRef

case class ActorContext(self: actor.ActorRef, ctx: actor.ActorContext)

class AkkaWrapper(var behavior: Behavior)
  extends actor.Actor with actor.ActorLogging {

  implicit val ctx = ActorContext(self, context)

  def receive = {
    case msg: Msg =>
      log.info(s"${behavior}: ${msg}")
      behavior = behavior.processMsg(msg)

    case _ => ()
  }
}

abstract class ActorSystem(val system: actor.ActorSystem) {
  def spawn(behavior: Behavior, name: String): actor.ActorRef = {
    system.actorOf(actor.Props(new AkkaWrapper(behavior)), name = name)
  }

  def run(): Unit
}
```

Listing 18: Akka shim for our Actor framework

4.8 Conclusion

In this section, we have presented a simple model for actor systems implemented as a PureScala library, and modeled a few simple systems with it. For each of these systems, we specified a global invariant which we subsequently proved was valid for some initial state and was preserved between each evaluation step. We have also shown how to run these systems on top of a Akka, a world-class actor implementation, thanks to a very shallow shim.

```

@extern
object System extends ActorSystem("rep-counter-sys") {
  def run(): Unit = {
    val backup = spawn(BackupB(0), "backup")
    val primary = spawn(PrimaryB(0, backup), "primary")

    primary ! Inc()
  }
}

@extern
def main(args: Array[String]): Unit = {
  System.run()
}

```

Listing 19: Akka shim for our Actor system framework

4.9 Further Work

Real-world Case Study

Admittedly, the systems we have modeled so far with this framework are quite small, and their complexity is far removed from that of a real-world system. While we have tried to verify more complex systems, we have not, in the time imparted to this project, been able to successfully prove the preservation of any interesting invariant. We do not believe that our model is fundamentally unadapted to bigger systems, but rather that more time would need to be allocated to such an endeavor. By contrast, the Verdi Coq framework, which is similar in spirit to our framework, has been successfully used to verify the entirety of a Coq implementation of the Raft consensus algorithm [16].

Weaker Guarantees On Message Delivery

As we mentioned in Section 4.3.1, our current model effectively implements and rely on *exactly-once* message delivery. Although such a guarantee is provided by real-world actor systems such as Akka when used as a basis for concurrency, it is not realistic to expect of a real-world distributed system implementation to rely on this same guarantee. We thus believe that the guarantees provided by our model ought to be somewhat weakened in order to be better suited to the modeling of distributed systems.

Name Uniqueness

It is important to note that, within this model, actor references are not guaranteed to be unique as a user could spawn two actors with the same name. It is therefore up to the user to ensure that no two spawned actors have the same name. To palliate to this problem, a possible solution would be to have the system maintain a counter specific to each actors, which would then be

both stored in child references, and subsequently incremented, in the style of the Actario Coq framework [17].

Integration with Partial Evaluator

We initially hoped that the partial evaluator we presented in Section 3 would help with the verification of these systems. Unfortunately, it appears that, in practice, the evaluator does not improve the verification time (quite the contrary, actually), nor does it remove any of the burden of the proof from the programmer. This might either be because this approach is not well suited to the problem at hand, or because of some limitations of the evaluator itself, which we hope to investigate further.

5 Bi-Party Communication Protocols

5.1 Motivation

Most systems made of components exchanging messages between them, such as Actor systems, do so by following a *communication protocol*. A protocol is a set of rules which the components must follow when receiving, processing, and replying to a message. While it is possible to verify that a system correctly implements a given protocol, doing so is usually not trivial and often requires a sizable amount of work, depending of course on the model one is working with. In the case of the actor model we developed in Section 4, in which messages are *untyped*, verifying that a system followed some kind of protocol often required encoding a small type system within the invariants, for example to ensure that a message sent by an actor was of the right type. Moreover, we also had to check whether an actor would actually send a reply when it was expected of it. While being very interesting properties to verify, we would rather rely on the actual type system to catch any mistakes directly when writing the implementation.

As we have unfortunately not found a way to provide a typed API for our model, and do not expect to find a solution within the scope of this project, we now turn our attention to simpler, synchronous systems involving only two parties communicating over a *channel* with two endpoints, one for sending messages and one for receiving them. Such a system might be an ATM and a user willing to withdraw money communicating through a screen, or a browser and a web server attempting to initialize a secure connection through a TCP socket. The protocols governing such systems that we are going to study in this chapter are called *bi-party communication protocols*. Such protocols have the interesting property that the description of the protocol from the viewpoint of one party is enough to deduce the protocol that must be followed from the viewpoint of the other party. The dialogue between two parties is commonly referred to as a *session*.

As a running example for the next sections, we will consider a very simple protocol involving two parties Alice and Bob, and four messages `Greet`, `Hello`, `Bye`, and `Quit`. From the viewpoint of Alice, the protocol is informally described as follows: Alice must send Bob either the message `Quit` or `Greet`. In the first case the sessions then ends, otherwise she can expect to receive either the message `Bye`, ending the session, or the message `Hello`, after which the session continues from the start.

For the an implementation of such protocols to be correct, assuming an underlying reliable delivery channel, each party must both send the correct message when it is expected of them, and handle all possible messages that they can expect to receive at some point. A implementation of a party which fails to do so is deemed incorrect in that framework.

5.2 Session Types

Session types [18] provide a way to encode such protocols at the type level, thus guiding the programmer during the implementation while also guarding against mistakes such as not handling a certain message type or failing to reply to a message. Their syntax is shown in Figure 3.

$T ::= S$		Int		Boolean		String		\dots	payload
$S ::= \&\{l_i : S_i\}_{i \in I}$									
		$\oplus\{l_i : S_i\}_{i \in I}$							branching
		$?T.S$							selection
		$!T.S$							receive
		$\mu_X.S$							send
		X							recursion
		ϵ							variable
		ϵ							termination

Figure 3: Syntax of session types

$$\begin{array}{lcl}
\overline{\overline{S}} & ::= & S \\
\overline{\&\{l_1 : S_1, \dots, l_n : S_n\}} & ::= & \oplus\{l_1 : \overline{S_1}, \dots, l_n : \overline{S_n}\} \\
\overline{?T.S} & ::= & !T.\overline{S} \\
\overline{\mu\alpha.S} & ::= & \mu\alpha.\overline{S[\overline{\alpha}/\alpha]} \\
\overline{X} & ::= & X \\
\overline{\epsilon} & ::= & \epsilon
\end{array}$$

Figure 4: Duality of session types

We show below the session types S_A and S_B corresponding to the protocol we defined in Section 5.1, from the viewpoint of Alice and Bob, respectively.

$$\begin{aligned}
S_A &= \mu\alpha. \left(!\text{Greet}. \left(?\text{Hello}.\alpha \& ?\text{Bye}.\epsilon \right) \oplus !\text{Quit}.\epsilon \right) \\
S_B &= \mu\alpha. \left(?\text{Greet}. \left(!\text{Hello}.\alpha \oplus !\text{Bye}.\epsilon \right) \& ?\text{Quit}.\epsilon \right)
\end{aligned}$$

We draw the reader's attention to the similarity between the two types, and note that the S_B reads the same as S_A if one substitutes ! for ?, \oplus for $\&$, and vice-versa. Each type is in fact the *dual* of the other, a property we formalize in Figure 4.

5.3 Session Types and Linearity

Although session types were originally meant to be implemented as a separate syntactic category of types and terms to be added to the π -calculus, it has been shown that it possible to encode them directly in a calculus or language with both *linear types* and *variants* [19,20].

While there are many ways to perform such an encoding and provide a collection of combinators to build values of such types while enforcing the associated safety properties, the existing approaches [21,22] either rely on more expressive type systems than the one provided by Stainless, eg. *substructural type systems* or ones which provide *path families*, *higher-kinded types* and *indexed monads*, or *path-dependent types*. In the next section we look at a solution to this problem which only require minimal and orthogonal modifications to Stainless’s type system.

5.4 Term-Level Sessions Encoding

The *lchannels* Scala library [23] provides a lightweight, term-level encoding of sessions which does not rely on advanced type system features. This encoding essentially corresponds to the *continuation-passing-style* transformation of session types. In their library, the two endpoints (one for receiving, one for sending) of the communication channel between two parties are represented as values of the two types `In` and `Out`, respectively. These types are parametrized by the type of value that they accept or produce, and provide methods to send or receive such messages. Listing 20 shows a subset the API provided by the library, and Listing 21 shows the value-level encoding of the protocol we have been working with so far with this framework, following the encoding described in section 7.3 of *Lightweight Session Programming in Scala* [23]. Of note is the fact that instead of having the dual type of the protocol computed statically at the type level, this property is enforced by the `!!` methods on the `Out` type, which effectively flips around the type of the channel, by forcing the consumer of the channel to make use of the returned channel, which has the dual type of the original, provided that the reference to the original channel cannot be re-used.

5.5 Linear Types in Stainless

In this section, we discuss our implementation⁷ of linear types in Stainless, which consists of about 750 lines of code. Because the PureScala AST we are working with in Stainless is already typed, there is no need to write a full-fledged type checker. We will hence rather describe a *linearity checker* for PureScala programs.

5.5.1 Introducing Linear Types

First of all, we need a way to introduce to mark some types as *linear*. To this end, we define a covariant type constructor `Linear`, which simply holds a value of type `A`. This type provides a `!` method to consume the linear term and return the underlying value. This enables the user to call a method of the underlying type in a concise way. As the astute reader might have noticed, this effectively adds weakening to the linear type system, and, as we will see, some care will be needed to handle such conversions properly. For example, if one had a value `foo` of type `Linear[Option[A]]`, one could call the `isEmpty` method on the underlying value by writing `foo!.isEmpty`. While making the consumption of a linear value explicit in this way is good for reasoning about one’s code, there is still a bit of clutter associated with it, we also introduce an opt-in implicit conversion `delinearize` from any `Linear[A]` to `A`. At last, because converting a non-linear value of type `A` to a linear value of type `Linear[A]` is always safe, we provide a such an

⁷<https://github.com/romac/stainless/tree/linear>


```

abstract class In[+A] {
  // Blocks until a message is received through the channel and returns it.
  def receive(implicit d: Duration): A

  // Map over the next message received and returns the result.
  def ?[B](f: A => B)(implicit d: Duration): B = {
    f(receive)
  }
}

abstract class Out[-A] {
  // Sends a message through the channel.
  def !(msg: A): Unit

  // Sends a message which will trigger a reply, and returns the
  // endpoint to receive the reply from.
  // Expects the provided function to embed the given
  // output endpoint into the message before returning it.
  // and returns an endpoint for receiving it.
  def !![B](h: Out[B] => A): In[B] = {
    val (in, out) = create[B]()
    this ! h(out)
    in
  }

  // Sends a message after which another message must be sent as well.
  def !![B](h: In[B] => A): Out[B]

  // Creates a new channel and return its two endpoints.
  def create[B](): (In[B], Out[B])
}

```

Listing 20: *lchannels* library interface

```

sealed abstract class AliceBob
case class Quit() extends AliceBob
case class Greet(cont: Out[Response]) extends AliceBob

sealed abstract class Response
case class Hello(cont: Out[AliceBob]) extends Response
case class Bye() extends Response

```

Listing 21: *lchannels* implementation of a simple protocol

implicit conversion by default, `linearize`. Listing 22 shows the full definitions. Because those will be extracted in a specific way, they are marked `@ignore`.

```

package stainless

import stainless.lang._
import stainless.annotation._

package object linear {

  @ignore
  class Linear[+A](_value: A) {
    def != _value
  }

  @ignore
  implicit def linearize[A](value: A): Linear[A] = new Linear(value)

  object implicits {
    @ignore
    implicit def delinearize[A](lin: Linear[A]): A = lin!
  }
}

```

Listing 22: Linear wrapper for Scala types and values

5.5.2 Preventing Weakening

We now describe what it means for a linear term to be *consumed*: a term t of type `Linear[A]`, for any type A , is deemed *consumed* in an expression e when any of the following propositions is true:

- The underlying value of type A is extracted, via the `!` method, eg. $e = t!$.
- The term is assigned to a variable, eg. `val s: Linear[A] = t`.
- The term is supplied as an argument to function, eg. given `def f(x: Linear[A]): B`, we have $e = f(t)$.
- The term is supplied as an argument to a method, eg. given a class C with a method `def m(x: Linear[A]): B`, a value $v : C$, we have $e = v.m(t)$.
- The term is supplied as an argument to a lambda, eg. given `val l: Linear[A] => B`, we have $e = l(t)$.
- The term is supplied as an argument to a constructor, eg. given `case class C(x: Linear[A])`, we have $e = C(t)$.

Note: Method calls are subsumed by the first rule, as linear terms must first be delinearized with the `!` operator before one can call methods on the underlying values.

We now must ensure that no linear term is *consumed* more than once. To this end, we must recursively walk down the AST, while keeping track of terms that have been consumed in a *usage context*, in order to disallow subsequent uses of those terms. We will denote this context by Δ .

The astute reader will have noticed that the presence of the `!` operator, if not handled carefully, would actually allow weakening. For example, given a value `a: Linear[A]`, one could write `val b: A = a.!`, and thus obtain a non-linear reference to the underlying value. To counter this, the linearity checker treats any expression of the form `e.!`, with `e: Linear[A]`, as having type `Linear[A]` instead of `A`.

5.5.3 Preventing Contraction

Because linear logic does not allow contraction, we must also ensure that no linear term is *dropped*, that is to say, that it is *consumed* at least once. To this end, we first collect all linear variables being introduced in a function definition, for example as a parameter to the function, in a variable binding, or within a pattern in a `clause` (even as a wildcard). Then, after having ran the type-checking algorithm described in the previous section, we can make use of the resulting *usage context* Δ to check whether each and every of those variable has indeed been *consumed* at some point, and reject the program otherwise.

5.5.4 Linear Terms in Contracts

It is important to note that, when running the linearity checker over a function with pre- and/or post-conditions, these are ignored for the following reason: a user might want to constrain either a linear parameter of some function, or its return value. If we ran the linearity checker on such contracts, then one would not be able to re-use the linear variable that is being constrained in the precondition, or would not be able to reference any linear parameter in the postcondition. Listing 23 shows such a use-case.

```
def foo(x: Linear[Option[BigInt]]): BigInt = {
  require(!x.isEmpty && x.get > 0)
  x.get * 2
} ensuring { _ > 0 }
```

Listing 23: Usage of a linear variable in a function’s precondition

Fortunately for us, because a function’s contract will be statically verified by Stainless, there is no point to check it at runtime. Hence, in Stainless’s library, both the `require` function and the `ensuring` method discard their body. For this reason, we can safely ignore linearity constraints in a function’s contract.

5.5.5 Linear Data Types

Because data types can contain linear fields, one must be careful as to when to allow values of such types to be introduced. Indeed, if one were to define a data type `case class A(x: Linear[B])`,

one should not be allowed to construct a non-linear term of type `A`. That is because doing so would permit the user to have more than one (indirect) reference to the linear `x` field, which is forbidden because of the No-Weakening rule. We must thus ensure that values such types are only introduced linearly, ie. as values of type `Linear[A]`, whether it is as a function parameter, as a variable binding, or as field of another data type. Listing 24 features a few examples of this rule in action.

```

case class A(x: Linear[BigInt])

case class B(a: A)           // error
case class C(b: Linear[A]) // ok

def f(x: A): A = { // error
  x
}

def g(x: Linear[A]): Linear[A] = { // ok
  x
}

def h(x: C): Linear[A] = c match {
  case C(b) => b // ok
}

def i(x: BigInt): A = c match { // error
  A(x)
}

def j(x: BigInt): Linear[A] = c match {
  A(x) // ok
}

def k(x: BigInt): Linear[BigInt] = c match {
  A(x) match {
    case A(y) => y // error
  }
}

```

Listing 24: Linear data types in action

5.5.6 Marking the Current Object as Linear

Another issue arise when dealing with data types meant to be introduced linearly if these have associated methods. To understand why, let's look at an example. Listing 25 shows a very simple API which allows to open a file, read its content line-by-line, or read all its content at once. We would like to make sure that, once a user opens a `File` and receives the associated `FileHandler`,

the latter must be closed. Unfortunately, when implementing the `contents` method, nothing prevents the programmer to call `readLine` twice on the current object. That is because, Within `contents`, `this` has type `FileHandler`, and calling any method on it will thus not consume it. As it is not possible in Scala to constrain the type of the current object, even with self-annotations, we introduce a method annotation `@linear` which signals to the linearity checker that, within an annotated method of a class `C`, `this` should be considered to have type `Linear[C]`. As methods inherit the annotations of their enclosing class, it is here enough to annotate the `FileHandler` class with `@linear`.

```
class File {
  def open: Linear[FileHandler] = /* ... */
}

class FileHandler {
  def readLine: (Option[String], Linear[FileHandler]) = /* ... */
  def close: Unit = /* ... */

  def contents: String = {
    val (res, h) = this.readLine
    doSomething(this.readLine) // should be disallowed
    res match {
      case Some(line) =>
        line + h!.contents

      case None() =>
        h!.close
        ""
    }
  }
}
```

Listing 25: Linear File API

5.5.7 Higher-Order Linear Functions

Listing 27 shows the syntactic sugar provided by our implementation, which allow to refer to the type a linear lambda, ie. a lambda which consumes its argument, by `A -> B` instead `Linear[A] => B`. The library also provides an implicit conversion from function of `Linear[A] => B` to a `A -> B`. In Listing ??, we show the definition of a linear list, along with its `map` method, which accepts a linear lambda.

5.6 Sessions Library in PureScala

Listing 28 shows a PureScala implementation of the `lchannels` library. For the purpose of verification, we do not need a full-fledged implementation of the channels, but only declarations

```

case class ->[A, B](f: Linear[A] => B) {
  def apply(x: Linear[A]): B = f(x)
}

implicit def toLin[A, B](f: Linear[A] => B): A -> B = ->(f)

```

Listing 26: Syntactic sugar for linear lambdas

```

@linear
sealed abstract class LList[A] {
  def map[B](f: A -> B): Linear[LList[B]] = this match {
    case LNil()      => LNil[B]()
    case LCons(h, t) => LCons(f(h), t.map(f))
  }
}

case class LNil[A]() extends LList[A]
case class LCons[A](head: Linear[A], tail: Linear[LList[A]]) extends LList[A]

def ok(xs: Linear[LList[Int]]): Linear[LList[Int]] = {
  xs.map((x: Linear[Int]) => x + 1)
}

def bad(xs: Linear[LList[Int]]): Linear[LList[Int]] = {
  xs.map((x: Linear[Int]) => x + x + 1) // err: linear term 'x' has already been used
}

```

Listing 27: Linear list

mirroring the Scala library. This way, one could run their implementation with the original library by simply linking against both it and the Stainless library, without relying on our library.

5.7 Case Study

Let's consider a protocol involving an ATM and its user, which we informally describe below:

- A The user authenticates herself by sending the ATM both her card number and PIN.
- B If the authentication succeeds, the ATM displays a menu to the user, who can then choose to:
 - (a) Abort the process altogether.
 - (b) Ask for her account's balance, in which case the server will reply with the balance, and displays the menu again.

```

type In[A] = Linear[InChan[A]]
type Out[A] = Linear[OutChan[A]]

@linear @library
class InChan[A] {

  @extern
  def receive(implicit d: Duration): Linear[A] = {
    ???
  }

  def ?[B](f: Linear[A] => B)(implicit d: Duration): B = {
    f(receive)
  }
}

@linear @library
class OutChan[A] {

  @extern
  def send(msg: A): Unit = {
    ???
  }

  def !(msg: A): Unit = {
    send(msg)
  }

  @extern
  def !![B](h: Out[B] => A): In[B] = {
    ???
  }

  @extern
  def create[B](): (In[B], Out[B]) = {
    ???
  }
}

```

Listing 28: Sessions library in PureScala

C If the authentication fails, the ATM notifies the user of the failure, and the process is aborted.

Listing 29 shows the encoding of such a specification using the library described in Section 5.6, while Listing 30 shows the corresponding valid implementation. At last, Listing 32 shows an

incorrect implementation of the protocol that would still verify without the linearity checker. We discuss the three use cases below:

1. If we provide an empty body for the `atm` function, we would then be greeted with the following error:

```
Error: linear variable 'c' of type 'Linear[In[Authenticate]]' is never used:
      def atm(c: Linear[In[Authenticate]]): Unit = {
          ^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
```

Re-using the same channel twice would also give rise to an error:

```
Error: linear term 'cont' has already been used: doSomething(cont)
          ^^^^
Info: term used here: cont !! Balance(balance(card))(_) ? menu(card)
                                     ^^^^
```

2. In case we forget to send back a failure notification when the authentication fails. The linearity checker will realise that the linear `cont` is not consumed in every branch of the pattern match, and will pinpoint its introduction:

```
Error: linear variable 'cont' of type 'Linear[OutChan[Response]]' is never used:
      case Authenticate(_, _, cont) =>
          ^^^^
```

3. At last, let's see what happens if we do not handle the reply to the `Success` message sent in case the authentication succeeds. Because the expression `cont !! Success(_)` has type `In[Menu]`, one could expect the Scala compiler to raise a type error, as the `atm` function has return type `Unit`. Unfortunately, the Scala compiler will happily convert any value to `Unit` if it occurs at the end of a block. But because `In[Menu]` is a linear type, the linearity checker will notice that the corresponding value is being discarded, and will raise an error:

```
Error: linear term cannot be discarded: cont !! Success(_)
          ^^^^^^^^^^^^^^^^^^^^^^^^^
```

5.8 Conclusion

In this section, we have presented and implemented a linear type system for PureScala, and put it to use in a term-level implementation of sessions types based on the `lchannels` library. This allowed to us ensure a safe usage of the library, effectively preventing a user from incorrectly implementing a bi-party communication protocol on top of a PureScala implementation of this library. Moreover, we have shown that no runtime errors resulting from the repeated use of linear channel can arise within that framework. Our implementation of the library is fully compatible with the original one, and can easily be swapped for the latter and then compiled using the


```

// Authentication request from the user
case class Authenticate(card: String, pin: String, cont: Out[Response])

// Authentication response from the ATM
sealed abstract class Response
case class Failure() extends Response
case class Success(cont: Out[Menu]) extends Response

// Choices available to authenticated user
sealed abstract class Menu
case class CheckBalance(cont: Out[Balance]) extends Menu
case class Quit() extends Menu

// User account balance
case class Balance(amount: BigInt)(cont: Out[Menu]) {
  require(amount >= 0)
}

```

Listing 29: ATM protocol description

standard Scala infrastructure, provided that one links the resulting program with the Stainless standard library.

5.9 Further Work

Term-level Multiparty Session Types for Actor Systems

We believe that it would be worth investigating the interplay between linear types, sessions types and the actor model developed in Section 4, provided that we manage to enrich the model with a typed API. It is of course already possible to mark actor references as linear within our model, effectively requiring the holder of such a reference to send it a message. But without a typed API, it is current not possible to model a session within this framework. On top of that, more work is probably needed to properly encode the resulting so-called *multiparty session types* [24] at the term level.

Affine Types and Borrow Checker

In the early days of this project, we implemented a very basic borrow checker within Stainless⁸. While the results were encouraging, such a mechanism is not enough to implement linear channels, as it corresponds to an enriched affine type system. We nonetheless believe that this could also be an interesting addition to Stainless. This because one could implement a pass within Stainless which would insert the appropriate invocations for allocating and freeing the memory associated with an affine term which could then be linked and compiled with the Scala Native project⁹, thus effectively allowing one to opt out of garbage collection for the parts of a program requiring

⁸<https://github.com/romac/stainless/tree/borrow-checker>

⁹<https://github.com/scala-native/scala-native>

```

def atm(c: In[Authenticate]): Unit = {
  c ? { auth => auth! match {
    case Authenticate(card, pin, cont) if authenticated(card, pin) =>
      cont !! Success(_) ? menu(card)

    case Authenticate(_, _, cont) =>
      cont ! Failure()
  } }
}

def menu(card: String)(menu: Linear[Menu]) = {
  menu! match {
    case CheckBalance(cont) =>
      cont !! Balance(balance(card))(_) ? menu(card)

    case Quit() => ()
  }
}

def authenticated(card: String, pin: String): Boolean = {
  /* ... */
}

def balance(card: String): BigInt = {
  /* ... */
} ensuring { _ >= 0 }

```

Listing 30: Correct ATM protocol implementation

```

def getAmount(c: Out[Authenticate], card: String, pin: String): Option[BigInt] = {
  c !! Authenticate(card, pin, _) ? { res =>
    res! match {
      case Failure() => None()
      case Success(cont) => cont !! CheckBalance(_) ? {
        case Balance(amount)(cont) =>
          cont ! Quit()
          Some(amount)
        }
      }
    }
  } ensuring { _ >= 0 }
}

```

Listing 31: Correct user protocol implementation

a high memory throughput. It is worth noting that this technique could also apply to Leon's

```

def atm(c: In[Authenticate]): Unit = {
  c ? { auth => auth! match {
    case Authenticate(card, pin, cont) if authenticated(card, pin) =>
      // 3. do not wait for a reply to 'Success' message
      cont !! Success(_)

    case Authenticate(_, _, cont) =>
      // 1. does not send back a Failure message

  } }
}

def menu(card: String)(menu: Linear[Menu]): Unit = {
  menu! match {
    case CheckBalance(cont) =>
      cont !! Balance(balance(card))(-) ? menu(card)

      // 2. 'cont' has already been used
      doSomething(cont)

    case Quit() => ()
  }
}

```

Listing 32: Incorrect implementation of the ATM protocol

C code generator [25,26] but as a way to safely introduce dynamic memory allocations in the generator, which currently only support statically allocated memory.

6 Conclusion

With this project, we hope to have demonstrated the versatility and extensibility of the Stainless verification system by showcasing three orthogonal approaches to the modeling and verification of non-trivial programs and systems. In Section 3 we presented the design and implementation of a partial symbolic evaluation procedure for PureScala programs, and argued with supporting examples, that this technique can enhance both the performance and the reasoning abilities of the verification system. In Section 4, we defined a model for actor systems, and verified in three case studies that some appropriate invariant was preserved during the execution of the system. At last, in Section 5, we have shown how the addition of linear types to the type system of Stainless allows for a safe encoding of session types in PureScala, inspired by the design of an existing Scala library.

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B Appendix

B.1 Implementation of an interpreter for a simple programming language

```
import stainless.lang._
import stainless.collection._
import stainless.annotation._
import stainless.util.Random

object comp {

  sealed trait Expr
  case class Var(name: String) extends Expr
  case class Num(value: Int) extends Expr
  case class Add(l: Expr, r: Expr) extends Expr
  case class Mul(l: Expr, r: Expr) extends Expr
  case class Rand(max: Expr) extends Expr

  case class Context(bindings: List[(String, Expr)]) {
    def apply(name: String): Option[Expr] = {
      bindings.find(_._1 == name).map(_._2)
    }
  }

  // case class Context(bindings: Map[String, Expr]) {
  //   def apply(name: String): Option[Expr] = {
  //     bindings.get(name)
  //   }
  // }

  implicit val state = Random.newState

  @extern
  def random(max: Int): Int = {
    Random.nextInt(max)
  }

  case class Error(msg: String)
  def interpret(expr: Expr, ctx: Context): Either[Error,
    ↪Int] = {
    expr match {
      case Num(value) => Right(value)

      case Var(name) => ctx(name) match {
        case None() => Left(Error("Unbound variable: "
          ↪ + name))
        case Some(value) => interpret(value, ctx)
      }

      case Add(l, r) => for {
        le <- interpret(l, ctx)
        re <- interpret(r, ctx)
      } yield le + re

      case Mul(l, r) => for {
        le <- interpret(l, ctx)
        re <- interpret(r, ctx)
      } yield le * re

      case Rand(max) =>
        interpret(max, ctx).map(random(_))
    }
  }
}
```

```

    }
  }

  val program: Expr = Mul(Num(10), Add(Var("x"), Rand(Num
    ↪(42))))

  @symeval
  def compiled(x: Int): Int = {
    interpret(program, Context(List("x" -> Num(x))))
    .get // }
    ↪ 10 * (ctx("x") + random(42))
  }

```

```

  }

  @symeval
  def test(y: Int) = {
    val ctx = Context(List("y" -> Num(y)))
    interpret(program, ctx)
  } ensuring { _.isLeft }

```

B.2 Implementation of an algebra over a key-value store

```

import stainless.lang._
import stainless.annotation._
import stainless.collection._

object kv {

  sealed abstract class Label
  object Label {
    case class Get(key: String) extends Label
    ↪Label
    case class Put(key: String, value: String) extends Label
    ↪Label
  }

  sealed abstract class Op
  case class Pure(value: Option[String]) extends Op
    ↪

```

```

    case class Get(key: String, next: Option[String] => Op)
    ↪ extends Op
    case class Put(key: String, value: String, next: () => Op)
    ↪ extends Op

  def get(key: String)(next: Option[String] => Op): Op =
    ↪ Get(key, next)
  def put(key: String, value: String)(next: () => Op): Op =
    ↪ Put(key, value, next)
  def pure(value: Option[String]): Op =
    ↪ Pure(value)

  def interpret(op: Op)(kv: Map[String, String], trace:
    ↪List[Label], n: BigInt): (Option[String], List[Label
    ↪]) = {
    require(n >= 0)
    decreases(n)

    op match {

```



```

    case Pure(value) =>
      (value, trace)

    case Get(key, next) if n > 0 =>
      interpret(next(kv.get(key)))(kv, Label.Get(key) ::
↪ trace, n - 1)

    case Put(key, value, next) if n > 0 =>
      interpret(next())(kv.updated(key, value), Label.Put
↪ (key, value) :: trace, n - 1)

    case _ =>
      (None(), trace)
  }
}

def insert(kvs: List[(String, String)])(after: Op): Op =
  ↪ kvs match {
    case Nil() => after
    case Cons((k, v), rest) => put(k, v) { () => insert(
↪ rest)(after) }
  }

val xs = List("foo" -> "bar", "toto" -> "tata")
val program = insert(xs) {

```

```

  get("foo") { foo =>
    pure(foo)
  }
}

@symeval
def result(map: Map[String, String], init: List[Label], n
↪ : BigInt): (Option[String], List[Label]) = {
  require(n > 10)
  interpret(program)(map, init, n)
} ensuring { rt => rt match {
  case (res, trace) => prop(res, trace)
} }

@inline
def prop(res: Option[String], trace: List[Label]) = {
  res == Some("bar") &&
  trace.take(3) == List(
    Label.Get("foo"),
    Label.Put("toto", "tata"),
    Label.Put("foo", "bar")
  )
}
}

```

B.3 Implementation of a linear list

```

import stainless.lang._
import stainless.annotation._
import stainless.linear._

```

```

object linlist {
  case class ->[A, B](f: Linear[A] => B) {

```

```

    def apply(x: Linear[A]): B = f(x)
  }

  implicit def toLinFun[A, B](f: Linear[A] => B): A -> B =
    ↪ ->(f)

  @linear
  sealed abstract class LList[A] {

    def map[B](f: A -> B): Linear[LList[B]] = this match {
      case LNil() =>
        val res: Linear[LList[B]] = LList.empty[B]
        res

      case LCons(h, t) =>
        val hb: Linear[B]          = f(h)
        val tb: Linear[LList[B]] = t.map(f)
        val res: Linear[LList[B]] = linearize[LList[B]](
          ↪ LCons(hb, tb))
        res
    }

    @inline
    def ::(x: Linear[A]): Linear[LList[A]] = {
      linearize[LList[A]](LCons(x, this))
    }
  }

```

```

  }
}

object LList {
  def empty[A]: Linear[LList[A]] = linearize[LList[A]](
    ↪ LNil[A]())
}

case class LNil[A]() extends LList[A]
case class LCons[A](head: Linear[A], tail: Linear[LList[A]
  ↪]) extends LList[A]

def ok(xs: Linear[LList[Int]]): Linear[LList[Int]] = {
  xs.map((x: Linear[Int]) => x + 1)
} ensuring { res => test(2) }

def bad(xs: Linear[LList[Int]]): Linear[LList[Int]] = {
  val f = (x: Linear[Int]) => x + x + 1
  xs.map(f)
} ensuring { res => test(2) }

def test(x: Int) = x > 0
}

```

B.4 Implementation of an actor system in PureScala

```

import stainless.lang._
import stainless.proof._
import stainless.collection._

```

```

import stainless.annotation._
import scala.language.postfixOps

```

```

import akka.actor

package object akkashim {

type ActorRef = actor.ActorRef

abstract class Msg

case class ActorContext(self: actor.ActorRef, ctx: actor.
    ↪ActorContext) {
  def spawn(behavior: Behavior, name: String): actor.
    ↪ActorRef = {
    ctx.actorOf(actor.Props(new Wrapper(behavior)), name =
    ↪name)
  }
}

abstract class Behavior {
  def processMsg(msg: Msg)(implicit ctx: ActorContext):
    ↪Behavior

package actors

import stainless.lang._
import stainless.collection._
import stainless.annotation._

import scala.language.postfixOps

object ActorSystem {
  def named(name: String): ActorSystem = {
    ActorSystem(
      name,

```

```

    @inline
    implicit def sameBehavior: Behavior = this
  }

object Behavior {
  case class Stopped() extends Behavior {
    def processMsg(msg: Msg)(implicit ctx: ActorContext):
      ↪Behavior = {
      Behavior.same
    }
  }

  @inline
  def same(implicit behavior: Behavior): Behavior =
    ↪behavior

  @inline
  def stopped: Stopped = Stopped()
}

    CMap(_ => Behavior.stopped),
    CMap(_ => List())
  )
}
}

case class ActorSystem(
  name: String,
  behaviors: CMap[ActorRef, Behavior],
  inboxes: CMap[(ActorRef, ActorRef), List[Msg]]
) {

```

```

def run(): Unit = {}

def step(from: ActorRef, to: ActorRef): ActorSystem = {
  inboxes(from -> to) match {
    case Nil() =>
      this

    case Cons(msg, msgs) =>
      val (newBehavior, toSend, toSpawn) = deliverMessage
      ↪(to, from, msg)

      val newBehaviors = updateBehaviors(toSpawn,
      ↪behaviors.updated(to, newBehavior))
      val newInboxes = updateInboxes(to, toSend, inboxes.
      ↪updated(from -> to, msgs))

      ActorSystem(
        name,
        newBehaviors,
        newInboxes
      )
  }
}

def updateInboxes(from: ActorRef, toSend: List[Packet],
  ↪inboxes: CMap[(ActorRef, ActorRef), List[Msg]]): CMap
  ↪[(ActorRef, ActorRef), List[Msg]] = toSend match {
  case Nil() =>
    inboxes
  case Cons(Packet(to, msg), tms) =>
    updateInboxes(from, tms, inboxes.updated(from -> to,
    ↪msg :: inboxes(from -> to)))
}

```

```

def updateBehaviors(toSpawn: List[(ActorRef, Behavior)],
  ↪behaviors: CMap[ActorRef, Behavior]): CMap[ActorRef,
  ↪Behavior] = toSpawn match {
  case Nil() =>
    behaviors
  case Cons((ref, behav), rbs) =>
    updateBehaviors(rbs, behaviors.updated(ref, behav))
}

def deliverMessage(to: ActorRef, from: ActorRef, msg: Msg
  ↪): (Behavior, List[Packet], List[(ActorRef, Behavior)
  ↪]) = {
  val behavior = behaviors(to)

  val ctx = ActorContext(to, Nil(), Nil())
  val nextBehavior = behavior.processMsg(msg)(ctx)

  (nextBehavior, ctx.toSend, ctx.toSpawn)
}

@inline
def isStopped(id: ActorRef): Boolean = {
  behaviors(id) == Behavior.stopped
}

def send(from: ActorRef, to: ActorRef, msg: Msg):
  ↪ActorSystem = {
  val inbox = msg :: inboxes(from -> to)
  ActorSystem(name, behaviors, inboxes.updated(from -> to
  ↪, inbox))
}

def withInbox(from: ActorRef, to: ActorRef, msgs: List[
  ↪Msg]): ActorSystem = {

```

```

    ActorSystem(name, behaviors, inboxes.updated(from -> to
    ↪, msgs))
}

def spawn(ref: ActorRef, behavior: Behavior): ActorSystem }
    ↪ = {

```

```

    ActorSystem(name, behaviors.updated(ref, behavior),
    ↪inboxes)
}

```

B.5 Examples of actor systems

```

package actors

import stainless.lang._
import stainless.proof._
import stainless.collection._
import stainless.annotation._

import scala.language.postfixOps

object counting {

  case class Primary() extends ActorRef("primary")
  case class Backup() extends ActorRef("backup")

  case class PrimBehav(counter: Counter) extends Behavior {

    override
    def processMsg(msg: Msg)(implicit ctx: ActorContext):
    ↪Behavior = msg match {
      case Inc =>
        Backup() ! Deliver(counter.increment)
        PrimBehav(counter.increment)

```

```

      case _ => Behavior.same
    }
  }

  case class BackBehav(counter: Counter) extends Behavior {

    override
    def processMsg(msg: Msg)(implicit ctx: ActorContext):
    ↪Behavior = msg match {
      case Deliver(c) =>
        BackBehav(c)

      case _ => Behavior.same
    }
  }

  case object Inc extends Msg
  case class Deliver(c: Counter) extends Msg

  case class Counter(value: BigInt) {

```

```

require(value >= 0)

@inline
def increment: Counter =
  Counter(value + 1)

@inline
def <=(that: Counter): Boolean = {
  this.value <= that.value
}
}

def isSorted(list: List[Msg]): Boolean = list match {
  case Nil() => true
  case Cons(Deliver(_), Nil()) => true
  case Cons(Deliver(Counter(a)), rest@Cons(Deliver(
    ↪Counter(b)), xs)) => a < b && isSorted(rest)
  case _ => false // we also reject if the list contains
    ↪other messages than Deliver
}

def validBehaviors(s: ActorSystem): Boolean = {
  s.behaviors(Primary()).isInstanceOf[PrimBehav] &&
  s.behaviors(Backup()).isInstanceOf[BackBehav]
}

def validRef(ref: ActorRef): Boolean = {
  ref == Primary() || ref == Backup()
}

def invariant(s: ActorSystem): Boolean = {
  forall((ref: ActorRef) => validRef(ref)) &&
  validBehaviors(s) &&
  s.inboxes(Primary() -> Primary()).isEmpty &&

```

```

s.inboxes(Backup() -> Backup()).isEmpty &&
s.inboxes(Backup() -> Primary()).isEmpty && {

  val PrimBehav(p) = s.behaviors(Primary())
  val BackBehav(b) = s.behaviors(Backup())
  val bInbox = s.inboxes(Primary() -> Backup())

  p.value >= b.value && isSorted(bInbox) && bInbox.
  ↪forall {
    case Deliver(Counter(i)) => p.value >= i
    case _ => true
  }
}

def theorem(s: ActorSystem, from: ActorRef, to: ActorRef)
  ↪: Boolean = {
  require(invariant(s) && validRef(from) && validRef(to))

  val newSystem = s.step(from, to)
  assert(lemma(s))
  invariant(newSystem)
} holds

def lemma_sameBehaviors(s: ActorSystem, from: ActorRef,
  ↪to: ActorRef): Boolean = {
  require(invariant(s) && validRef(from) && validRef(to))

  assert(validBehaviors(s.step(Primary(), Backup())))
  validBehaviors(s.step(from, to))
} holds

def lemma(s: ActorSystem): Boolean = {
  require(invariant(s))

```

```

s.inboxes(Primary() -> Backup()) match {
  case Nil() => s.step(Primary(), Backup()) == s
  case Cons(Deliver(c), rest) =>
    assert(lemma_sameBehaviors(s, Primary(), Backup())) }
  val BackBehav(b) = s.step(Primary(), Backup()).
  ↪behaviors(Backup())

```

```
package actors
```

```

import stainless.lang._
import stainless.proof._
import stainless.collection._
import stainless.annotation._

```

```
import scala.language.postfixOps
```

```
object replicated {
```

```

  case class Primary() extends ActorRef("primary", None())
  case class Backup() extends ActorRef("backup", None())

```

```
  case class PrimBehav(counter: Counter) extends Behavior {
```

```

    override
    def processMsg(msg: Msg)(implicit ctx: ActorContext):
    ↪Behavior = msg match {
      case Inc =>
        Backup() ! Inc
        PrimBehav(counter.increment)
    }
  }

```

```
}
```

```

    b.value == c.value
  }
} holds

```

```
case class BackBehav(counter: Counter) extends Behavior {
```

```
  override
```

```
  def processMsg(msg: Msg)(implicit ctx: ActorContext):
```

```
  ↪Behavior = msg match {
```

```
    case Inc =>
```

```
      BackBehav(counter.increment)
```

```
  }
```

```
}
```

```
case object Inc extends Msg
```

```

case class Counter(value: BigInt) {
  require(value >= 0)

```

```
  @inline
```

```
  def increment: Counter =
    Counter(value + 1)
```

```
  @inline
```

```
  def <=(that: Counter): Boolean = {
    this.value <= that.value
  }
```

```
}
```

```
@inline
```

```

def noMsgToSelf(s: ActorSystem, ref: ActorRef): Boolean =
  ↪ {
    s.inboxes(ref -> ref).isEmpty
  }

def invariant(s: ActorSystem): Boolean = {
  noMsgToSelf(s, Backup()) && {
    (s.behaviors(Primary()), s.behaviors(Backup())) match
  ↪ {
    case (PrimBehav(p), BackBehav(b)) =>
      p.value == b.value + s.inboxes(Primary() ->
  ↪Backup()).length

    case _ => false
  }
}

package actors

import stainless.lang._
import stainless.proof._
import stainless.collection._
import stainless.annotation._

import scala.language.postfixOps

object spawn {

  case class Primary() extends ActorRef("primary")

  case class BeforeB() extends Behavior {
    def processMsg(msg: Msg)(implicit ctx: ActorContext):
  ↪Behavior = msg match {
      case Spawn() =>

```



```

case class Spawn() extends Msg

val childRef = Child("child", Primary())

def invariant(s: ActorSystem): Boolean = {
  s.behaviors(Primary()) match {
    case BeforeB() =>
      s.isStopped(childRef)

    case AfterB(child) =>
      child == childRef &&
      s.behaviors(childRef) == ChildB()
  }
}

case _ => false
}

def theorem(s: ActorSystem, from: ActorRef, to: ActorRef)
  ⇨: Boolean = {
  require(invariant(s))
  invariant(s.step(from, to))
} holds
}

```

B.6 Shim for running an actor system on top of Akka

```

import stainless.lang._
import stainless.proof._
import stainless.collection._
import stainless.annotation._

import scala.language.postfixOps

import akka.actor

package object akkashim {

  type ActorRef = actor.ActorRef

  abstract class Msg

  case class ActorContext(self: actor.ActorRef, ctx: actor.
    ⇨ActorContext) {
    def spawn(behavior: Behavior, name: String): actor.
      ⇨ActorRef = {
      ctx.actorOf(actor.Props(new Wrapper(behavior)), name =
        ⇨name)
    }
  }

  abstract class Behavior {
    def processMsg(msg: Msg)(implicit ctx: ActorContext):
      ⇨Behavior

    @inline
    implicit def sameBehavior: Behavior = this
  }
}

```

```

object Behavior {
  case class Stopped() extends Behavior {
    def processMsg(msg: Msg)(implicit ctx: ActorContext):
      ↪ Behavior = {
        Behavior.same
      }
  }
}

@inline
def same(implicit behavior: Behavior): Behavior =
  ↪ behavior

@inline
def stopped: Stopped = Stopped()
}

class Wrapper(var behavior: Behavior) extends actor.Actor
  ↪ with actor.ActorLogging {

  implicit val ctx = ActorContext(self, context)

  def receive = {

```

```

    case msg: Msg =>
      log.info(s"behavior :msg")
      behavior = behavior.processMsg(msg)
    case _ => ()
  }
}

abstract class ActorSystem(val name: String) {

  lazy val system = actor.ActorSystem(name)

  def spawn(behavior: Behavior, name: String): actor.
    ↪ ActorRef = {
      system.actorOf(actor.Props(new Wrapper(behavior)), name
    ↪ = name)
  }

  def run(): Unit
}
}

```